SOLID STATE PHYSICS 1

- November 2006 -

Do not forget to write your full name and student number on each sheet.

Please use separate sheets for each of the four problems.

Planck's constant	$\hbar$	1.055×10^{-34} Js
Planck's constant	h	6.626×10^{-27} erg s
Boltzmann's constant	k_B	1.38×10^{-23} J K^{-1}
Permittivity of free space	ε_0	8.85×10^{-12} F m^{-1}
Permeability of free space	μ_0	$4\pi \times 10^{-7}$ H m^{-1}
speed of light	c	3.0×10^{8} m s^{-1}
elementary charge	e	1.60×10^{-19} C
mass of the electron	m	9.11×10^{-31} kg
Bohr magneton	μ_B	9.27x10-24 J T^{-1}
Note also: 1 eV = 1.6019×10^{-12} erg		

I. The following questions should be answered very briefly (2-3 sentences at most); [2 points for each correct response].

a) What defines a perfect solid? (List at least four of the properties of a perfect solid.)

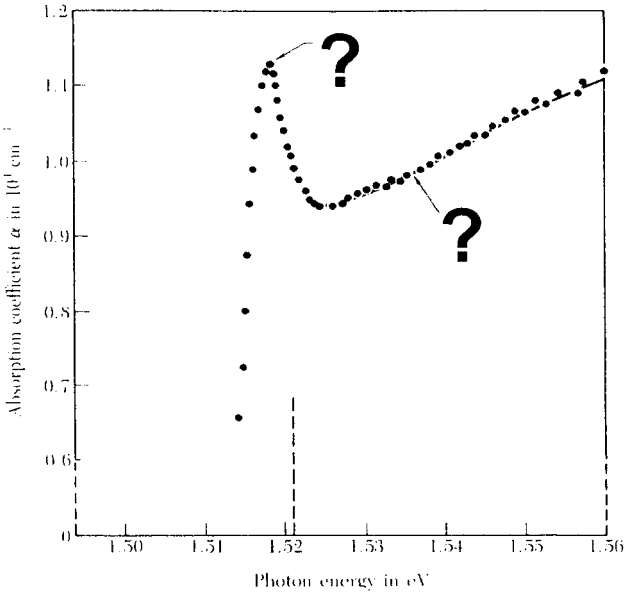
b) What determines the electrical conductivity of a semiconductor at low and at high temperatures ?

c) In a metal, what is a plasmon? How can it be excited?

d) What is thermopower? What can we learn from it regarding the charge carriers in a semiconductor?

e) What is the Curie temperature? What phases does it separate?

f) Explain the features indicated by "?" in the optical absorption spectrum of GaAs taken at 21K.



g) What is the necessary condition for a structural phase transition to occur in a crystal?

h) What does the excitation of a magnon correspond to?

i) Which of the following are assumptions of the Drude model?

I. when electrons undergo collisions, it results in a gradual velocity change;
II. collisions occur with a probability per unit time $1/\tau$;
III. electrons collide only with electrons;
IV. electrons collide only with ion cores;
V. between collisions, electrons do not interact with each other;
VI. between collisions, electrons do not interact with ion cores;
VII. the hotter the region in which a collision occurs, the faster a typical electron will emerge from the collision.

j) When Nb is cooled below 9,5 K the conduction electrons go from a disordered to an ordered state. What is this ordered state called? What is the nature of the ordering? What happens to the resistivity of Nb?

II. (a) What determines the diffraction pattern and the diffraction intensities in X-ray scattering? What do the X-rays interact with? [3 points]

The non-existing compound KnBy has a simple cubic lattice with a basis consisting of a Kn^+ ion at coordinate (0,0,0) and a By^- ion at (1/2, 1/2, 1/2). The lattice parameter is $a=5.05Å$. The form factors of Kn^+ and By^- have the ratio $f_{Kn}/f_{By} = 3/2$.

(b) Determine for which reciprocal lattice vectors $\mathbf{G} = v_1\mathbf{b}_1 + v_2\mathbf{b}_2 + v_2\mathbf{b}_2$ reflections will be observable in the X-ray diffraction spectrum. [2 points]

(c) Calculate the intensity $I = |S(v_1,v_2,v_3)|^2$ of these diffraction spots. For which values of (v_1,v_2,v_3) is the intensity high, and for which ones low? [3 points]

Hint: The structure factor is $S(\mathbf{G}) = \Sigma_j f_j \exp(-i\mathbf{G}.\mathbf{r}_j)$.

$\mathbf{r}_j$ is the position of the j-th atom in the conventional unit cell.

(d) If you use neutrons instead of X-rays, what other properties of the KnBy crystal apart from the structure can you determine and why? [2 points]

III. (a) Explain what is meant by a phonon, and how they are useful for considering the dynamics and thermal properties of crystals. **No formula, just description.** [2 points].

(b) Discuss the interactions that are possible between phonons, and what causes them [2 points].

The dispersion relation of the phonons in a two dimensional system is given by:

$$\omega_\kappa^2 = \frac{4C}{m}\left(sin^2\frac{k_x a}{2} + sin^2\frac{k_y b}{2}\right)$$

Where a and b are the lattice parameters of the system.

(c) Draw the dispersion ω_κ from the point $(k_x, k_y) = (0,0)$ to $(\pi/a\ ,\ 0)$, then from $(\pi/a\ ,\ 0)$ to $(\pi/a\ ,\ 2\pi/b)$ and then from $(\pi/a\ ,\ 2\pi/b)$ to $(2\pi/a\ ,\ 2\pi/b)$. [3 points]

(d) What is the physical meaning of the first Brillouin zone in relation to the phonon dispersion? [1 point]

(e) Calculate the group velocity for $|k| \to 0$ along the direction for which $k_x = k_y$. [1 point].

(f) What is the physical meaning of this group velocity if ω_κ represents longitudinal waves? [1 point]

IV. (a) Titanium oxide (TiO_2) is an ionic insulator. What are the charge and the electronic configuration of the Ti and O ions in this solid? [1 point]

(b) What is the magnetic behaviour of TiO_2? [1 point]

(c) Suppose that we dope TiO_2 with a small amount of vanadium (V), which replaces some of the Ti ions. Assume that, because O likes to have a filled shell, the valence state of the V ions is the same as that of the Ti atoms, and that except for the electrons that V gives up to the oxygen, all others stay on their respective ions. Explain why the material remains an insulator. [1 point]

(d) Due to the crystal field splitting, the orbital momentum of the V ions is quenched. Explain how this comes about, and what it means for the L_z, S_z and J_z of the 3d shell of a V ion. [2 points]

(e) We now apply a magnetic field. What is the dominant magnetic response of V-doped TiO_2 to this applied field? [1 point]

(f) Calculate the susceptibility as a function of temperature, ignoring all diamagnetic contributions and assuming $\boldsymbol{\mu}\cdot\mathbf{B} \ll k_B T$. If you don't know how to calculate it, state at least what you expect for its T-dependence. [4 points]